Review Problems

April 5, 2017

1. (Fall 2006, Exam 3, #8) Let $f(x)$ be the function which is represented by the power series
$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{n^3}.$$
Find the fifth derivative of the function f at $x = 1$.

2. (Fall 2007, Exam 3, #11) Let $f(x)$ be a function such that $f'(x) = x^2 \cos x$ and that $f(0) = 0$. What is the Maclaurin series of $f(x)$?

3. (Fall 2002, Exam 3, #7) Find the power series expansion for the function $\frac{d}{dx}\left(\frac{1}{1+x^2}\right)$.

4. (Fall 2002, Exam 3, #8) Represent $\int_0^1 \cos(x^2)dx$ as an infinite series.

5. (Fall 2002, Exam 3, #9) If $\ln(x)$ is expanded as a power series of the form $\sum_{n=0}^{\infty} c_n(x-3)^n$, what is c_3?

6. (Fall 2002, Exam 3, #10) Find the first three terms of the Maclaurin series of $f(x) = \sqrt[3]{8+4x}$.

7. (Fall 2006, Exam 3, #7) What is the interval of convergence of the power series
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2(x-2)^n}{3^n(n^3+2)}?$$

8. (Fall 2006, Exam 3, #10) Find the Taylor series of $f(x) = \dfrac{1}{5-x}$ centered at $a = 1$.

9. (Fall 2006, Exam 3, #11) Recall that $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, $-\infty < x < \infty$. Use this to find the Maclaurin series of
$$\int x^2 \sin x \, dx.$$